１４２．定義に従って微分②

導関数の定義に従って，次の関数を微分せよ。

(1) $f(x)=\dfrac{1}{x^2}$

$$f'(x)=\lim_{h\to 0}\frac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}=\lim_{h\to 0}\frac{x^2-(x+h)^2}{hx^2(x+h)^2}=\lim_{h\to 0}\frac{-2hx-h^2}{hx^2(x+h)^2}=\lim_{h\to 0}\frac{-2x-h}{x^2(x+h)^2}=\frac{-2x}{x^4}=-\frac{2}{x^3}$$

(2) $f(x)=\sqrt{x}$

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=\lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$=\frac{1}{2\sqrt{x}}$$

(3) $f(x)=\sin x$

$$f'(x)=\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}=\lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$$

$$=\lim_{h\to 0}\left\{\frac{\sin x(\cos h-1)}{h}+\cos x\cdot\frac{\sin h}{h}\right\}=\lim_{h\to 0}\left\{\frac{\sin x(\cos^2 h-1)}{h(\cos h+1)}+\cos x\cdot\frac{\sin h}{h}\right\}$$

$$=\lim_{h\to 0}\left\{-\frac{\sin x\sin^2 h}{h(\cos h+1)}+\cos x\cdot\frac{\sin h}{h}\right\}=\lim_{h\to 0}\left\{-\left(\frac{\sin h}{h}\right)^2\cdot\frac{h\sin x}{\cos h+1}+\cos x\cdot\frac{\sin h}{h}\right\}$$

$$=1^2\cdot\frac{0}{1+1}+\cos x\cdot 1=\cos x$$

(4) $f(x)=\log x$

$$f'(x)=\lim_{h\to 0}\frac{\log(x+h)-\log x}{h}=\lim_{h\to 0}\frac{\log\dfrac{x+h}{x}}{h}=\lim_{h\to 0}\frac{1}{h}\log\left(1+\frac{h}{x}\right)=\lim_{h\to 0}\frac{1}{x}\cdot\frac{x}{h}\log\left(1+\frac{h}{x}\right)$$

$$=\lim_{h\to 0}\frac{1}{x}\log\left(1+\frac{h}{x}\right)^{\frac{x}{h}}=\frac{1}{x}\log e=\frac{1}{x}$$