# １３９．数列の極限②

(1) $\dfrac{1}{2}$　(2) $-\dfrac{1}{2}$　(3) $\dfrac{1}{2}$

(4) $0<a\leqq 1$のとき　$\infty$，　$1<a<3$のとき　$\infty$，　$a=3$のとき　$\dfrac{2}{3}$，　$a>3$のとき　$\dfrac{1}{3}$

次の数列の極限を求めよ。

(1) $\dfrac{1+2+\cdots\cdots+n}{n^2}=\dfrac{\frac{1}{2}n(n+1)}{n^2}=\dfrac{1}{2}\left(1+\dfrac{1}{n}\right)$

よって $\displaystyle\lim_{n\to\infty}\frac{1+2+\cdots\cdots+n}{n^2}=\lim_{n\to\infty}\frac{1}{2}\left(1+\frac{1}{n}\right)=\frac{1}{2}$

(2) $\dfrac{1+2+\cdots\cdots+n}{n+2}-\dfrac{n}{2}=\dfrac{\frac{1}{2}n(n+1)}{n+2}-\dfrac{n}{2}=\dfrac{1}{2}\left\{\dfrac{n(n+1)}{n+2}-n\right\}=\dfrac{1}{2}\left(\dfrac{-n}{n+2}\right)=\dfrac{1}{2}\left(\dfrac{-1}{1+\frac{2}{n}}\right)$

よって $\displaystyle\lim_{n\to\infty}\left(\frac{1+2+\cdots\cdots+n}{n+2}-\frac{n}{2}\right)=\lim_{n\to\infty}\frac{1}{2}\left(\frac{-1}{1+\frac{2}{n}}\right)=-\frac{1}{2}$

(3) $\dfrac{1\cdot1+2\cdot3+3\cdot5+\cdots+n(2n-1)}{1^2+2^2+3^2+\cdots+(2n)^2}=\dfrac{\sum_{k=1}^{n}k(2k-1)}{\sum_{k=1}^{n}(2k)^2}=\dfrac{2\cdot\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)}{4\cdot\frac{1}{6}n(n+1)(2n+1)}=\dfrac{1}{2}-\dfrac{3}{4}\cdot\dfrac{1}{2n+1}$

よって $\displaystyle\lim_{n\to\infty}\left(\frac{1+2+\cdots\cdots+n}{n+2}-\frac{n}{2}\right)=\lim_{n\to\infty}\left(\frac{1}{2}-\frac{3}{4}\cdot\frac{1}{2n+1}\right)=\frac{1}{2}$

(4) $\dfrac{a^n+3^n}{a^{n+1}+1}$（$a$は正の定数）

(ｉ) $0<a\leqq 1$のとき　$\displaystyle\lim_{n\to\infty}\frac{a^n+3^n}{a^{n+1}+1}=\infty$

(ⅱ) $1<a<3$のとき　$\displaystyle\lim_{n\to\infty}\frac{a^n+3^n}{a^{n+1}+1}=\lim_{n\to\infty}\frac{\left(\frac{a}{3}\right)^n+1}{a\left(\frac{a}{3}\right)^n+\frac{1}{3^n}}=\infty$

(ⅲ) $a=3$のとき　$\displaystyle\lim_{n\to\infty}\frac{a^n+3^n}{a^{n+1}+1}=\lim_{n\to\infty}\frac{2\cdot3^n}{3^{n+1}+1}=\lim_{n\to\infty}\frac{2}{3+\frac{1}{3^n}}=\frac{2}{3}$

(ⅳ) $a>3$のとき　$\displaystyle\lim_{n\to\infty}\frac{a^n+3^n}{a^{n+1}+1}=\lim_{n\to\infty}\frac{1+\left(\frac{3}{a}\right)^n}{a+\frac{1}{a^n}}=\frac{1}{3}$