１３８．数列の極限①

(1) ∞	(2) $\frac{5}{4}$	(3) 0	(4) ∞	(5) 1	(6) $\frac{1}{2}$	(7) $\frac{3}{2}$	(8) 0

次の数列の極限を求めよ。

(1) n^2-n

$$\lim_{n\to\infty}\left(n^2-n\right)=\lim_{n\to\infty}n^2\left(1-\frac{1}{n}\right)=\infty$$

(2) $\dfrac{5n^2+3n+2}{4n^2+2n-1}$

$$\lim_{n\to\infty}\frac{5n^2+3n+2}{4n^2+2n-1}=\lim_{n\to\infty}\frac{n^2\left(5+\frac{3}{n}+\frac{2}{n^2}\right)}{n^2\left(4+\frac{2}{n}-\frac{1}{n^2}\right)}=\lim_{n\to\infty}\frac{5+\frac{3}{n}+\frac{2}{n^2}}{4+\frac{2}{n}-\frac{1}{n^2}}=\frac{5}{4}$$

(3) $\dfrac{3n-4}{n^2+1}$

$$\lim_{n\to\infty}\frac{3n-4}{n^2+1}=\lim_{n\to\infty}\frac{n^2\left(\frac{3}{n}-\frac{4}{n^2}\right)}{n^2\left(1+\frac{1}{n^2}\right)}=\lim_{n\to\infty}\frac{\frac{3}{n}-\frac{4}{n^2}}{1+\frac{1}{n^2}}=0$$

(4) $\dfrac{4^{n+1}}{2^{n+1}+3^n}$

$$\lim_{n\to\infty}\frac{4^{n+1}}{2^{n+1}+3^n}=\lim_{n\to\infty}\frac{4^n\cdot 4}{4^n\left\{2\cdot\left(\frac{1}{2}\right)^n+\left(\frac{3}{4}\right)^n\right\}}=\lim_{n\to\infty}\frac{4}{2\cdot\left(\frac{1}{2}\right)^n+\left(\frac{3}{4}\right)^n}=\infty$$

(5) $\dfrac{3^{2n}-6^{n+1}}{2^{2n}+9^n}$

$$\lim_{n\to\infty}\frac{3^{2n}-6^{n+1}}{2^{2n}+9^n}=\lim_{n\to\infty}\frac{9^n-6\cdot 6^n}{4^n+9^n}=\lim_{n\to\infty}\frac{9^n\left\{1-6\left(\frac{2}{3}\right)^n\right\}}{9^n\left\{\left(\frac{4}{9}\right)^n+\right\}1}=\lim_{n\to\infty}\frac{1-6\left(\frac{2}{3}\right)^n}{\left(\frac{4}{9}\right)^n+1}=1$$

(6) $\sqrt{n^2+2n}-\sqrt{n^2+n}$

$$\lim_{n\to\infty}\left(\sqrt{n^2+2n}-\sqrt{n^2+n}\right)=\lim_{n\to\infty}\frac{\sqrt{n^2+2n}-\sqrt{n^2+n}}{1}\cdot\frac{\sqrt{n^2+2n}+\sqrt{n^2+n}}{\sqrt{n^2+2n}+\sqrt{n^2+n}}$$

$$=\lim_{n\to\infty}\frac{n^2+2n-(n^2+n)}{\sqrt{n^2+2n}+\sqrt{n^2+n}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{2}{n}}+\sqrt{1+\frac{1}{n}}}=\frac{1}{2}$$

(7) $\sqrt{n+1}\left(\sqrt{n+2}-\sqrt{n-1}\right)$

$$\lim_{n\to\infty}\sqrt{n+1}\left(\sqrt{n+2}-\sqrt{n-1}\right)=\lim_{n\to\infty}\sqrt{n+1}\left(\frac{\sqrt{n+2}-\sqrt{n-1}}{1}\cdot\frac{\sqrt{n+2}+\sqrt{n-1}}{\sqrt{n+2}+\sqrt{n-1}}\right)$$

$$=\lim_{n\to\infty}\frac{3\sqrt{n+1}}{\sqrt{n+2}+\sqrt{n-1}}=\lim_{n\to\infty}\frac{3\sqrt{1+\frac{1}{n}}}{\sqrt{1+\frac{2}{n}}+\sqrt{1-\frac{1}{n}}}=\frac{3}{2}$$

(8) $\frac{1}{n}\cos\frac{n\pi}{4}$

$-1\leqq\cos\frac{n\pi}{4}\leqq1$ より $-\frac{1}{n}\leqq\cos\frac{n\pi}{4}\leqq\frac{1}{n}$

ここで，$\lim_{n\to\infty}\left(-\frac{1}{n}\right)=\lim_{n\to\infty}\frac{1}{n}=0$ であるから

はさみうちの原理より $\lim_{n\to\infty}\frac{1}{n}\cos\frac{n\pi}{4}=0$