１３０．ド・モアブルの定理

（1）-1　（2）$-i$　（3）-64　（4）$\dfrac{-81\sqrt{3}+81}{2}$　（5）-1　（6）$32i$

次の式を計算せよ。

(1) $\left(\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)^4$

$$\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^4=\cos\left(\frac{\pi}{4}\cdot 4\right)+i\sin\left(\frac{\pi}{4}\cdot 4\right)=\cos\pi+i\sin\pi=-1+i\cdot 0=-1$$

(2) $\left(\cos\dfrac{\pi}{6}-i\sin\dfrac{\pi}{6}\right)^3$

$$\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^3=\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^3=\left(\cos\frac{11}{6}\pi+i\sin\frac{11}{6}\pi\right)^3=\cos\left(\frac{11}{6}\pi\cdot 3\right)+i\sin\left(\frac{11}{6}\pi\cdot 3\right)$$

$$=\cos\frac{11}{2}\pi+i\sin\frac{11}{2}\pi=\cos\frac{3}{2}\pi+i\sin\frac{3}{2}\pi=0+i\cdot(-1)=-i$$

(3) $(1+i)^{12}$

$$(1+i)^{12}=\left\{\sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)\right\}^{12}=\sqrt{2}^{12}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^{12}=64\left\{\cos\left(\frac{\pi}{4}\cdot 12\right)+i\sin\left(\frac{\pi}{4}\cdot 12\right)\right\}^{12}$$

$$=64\left(\cos 3\pi+i\sin 3\pi\right)=64(-1+i\cdot 0)=-64$$

(4) $\left(\dfrac{3-\sqrt{3}i}{2}\right)^8$

$$\left(\frac{3-\sqrt{3}i}{2}\right)^8=\left(\frac{3}{2}-\frac{\sqrt{3}}{2}i\right)^8=\left\{\sqrt{3}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)\right\}^8=\sqrt{3}^8\left(\cos\frac{11}{6}\pi+i\sin\frac{11}{6}\pi\right)^8$$

$$=81\left\{\cos\left(\frac{11}{6}\pi\cdot 8\right)+i\sin\left(\frac{11}{6}\pi\cdot 8\right)\right\}=81\left(\cos\frac{44}{3}\pi+i\sin\frac{44}{3}\pi\right)$$

$$=81\left(\cos\frac{2}{3}\pi+i\sin\frac{2}{3}\pi\right)=81\left(-\frac{\sqrt{3}}{2}+i\cdot\frac{1}{2}\right)=\frac{-81\sqrt{3}+81}{2}$$

(5) $\left(\dfrac{\sqrt{3}-i}{\sqrt{3}+i}\right)^3$

$$\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^3=\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\cdot\frac{\sqrt{3}-i}{\sqrt{3}-i}\right)^3=\left(\frac{2-2\sqrt{3}i}{4}\right)^3=\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^3=\left(\cos\frac{5}{3}\pi+i\sin\frac{5}{3}\pi\right)^3$$

$$=\cos\left(\frac{5}{3}\pi\cdot 3\right)+i\sin\left(\frac{5}{3}\pi\cdot 3\right)=\cos 5\pi+i\sin 5\pi=-1$$

(6) $\left(\dfrac{5-i}{2-3i}\right)^{10}$

$$\left(\frac{5-i}{2-3i}\right)^{10}=\left(\frac{5-i}{2-3i}\cdot\frac{2+3i}{2+3i}\right)^{10}=\left(\frac{13+13i}{13}\right)^{10}=(1+i)^{10}=\left\{\sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)\right\}^{10}=\sqrt{2}^{10}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^{10}$$

$$=32\left\{\cos\left(\frac{\pi}{4}\cdot 10\right)+i\sin\left(\frac{\pi}{4}\cdot 10\right)\right\}=32\left(\cos\frac{5}{2}\pi+i\sin\frac{5}{2}\pi\right)=32(0+i\cdot 1)=32i$$