

$$(1) \text{ (i) } 5 \quad \text{(ii) } 8 \quad \text{(iii) } 5 \quad \text{(iv) } \frac{\sqrt{2}}{2}$$

$$(2) \text{ (i) } \frac{\sqrt{2}}{2} \left( \cos \frac{7}{4} \pi + i \sin \frac{7}{4} \pi \right) \quad \text{(ii) } 2 \left( \cos \frac{11}{6} \pi + i \sin \frac{11}{6} \pi \right)$$

$$\text{(iii) } \cos \frac{4}{3} \pi + i \sin \frac{4}{3} \pi \quad \text{(iv) } 2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

次の問いに答えよ。

(1) 次の複素数の絶対値を求めよ。

$$\text{(i) } -3+4i$$

$$|-3+4i| = \sqrt{(-3)^2 + 4^2} = \sqrt{25} = 5$$

$$\text{(ii) } 8i$$

$$|8i| = 8$$

$$\text{(iii) } (1-2i)^2$$

$$(1-2i)^2 = 1-4i+4i^2 = -3-4i \text{ より}$$

$$|-3-4i| = \sqrt{(-3)^2 + (-4)^2} = \sqrt{25} = 5$$

$$\text{(iv) } \frac{2+3i}{5-i}$$

$$\frac{2+3i}{5-i} \cdot \frac{5+i}{5+i} = \frac{7+17i}{26} \text{ より}$$

$$\left| \frac{7+17i}{26} \right| = \sqrt{\left( \frac{7}{26} \right)^2 + \left( \frac{17}{26} \right)^2} = \frac{\sqrt{49+289}}{26} = \frac{\sqrt{338}}{26} = \frac{13\sqrt{2}}{26} = \frac{\sqrt{2}}{2}$$

(2) 次の複素数を極形式で表せ。偏角  $\theta$  は  $0 \leq \theta < 2\pi$  とする。

$$\text{(i) } \frac{4+3i}{1+7i}$$

$$\frac{4+3i}{1+7i} \cdot \frac{1-7i}{1-7i} = \frac{25-25i}{50} = \frac{1}{2} - \frac{1}{2}i$$

$$\sqrt{\left( \frac{1}{2} \right)^2 + \left( -\frac{1}{2} \right)^2} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \text{ であるから}$$

$$\frac{1}{2} - \frac{1}{2}i = \frac{\sqrt{2}}{2} \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right) = \frac{\sqrt{2}}{2} \left( \cos \frac{7}{4} \pi + i \sin \frac{7}{4} \pi \right)$$

$$(ii) \sqrt{3} + \frac{1-i}{1+i}$$

$$\sqrt{3} + \frac{1-i}{1+i} \cdot \frac{1-i}{1-i} = \sqrt{3} + \frac{-2i}{2} = \sqrt{3} - i$$

$$\sqrt{\sqrt{3}^2 + (-1)^2} = 2 \text{ であるから}$$

$$\sqrt{3} + \frac{1-i}{1+i} = 2 \left( \frac{\sqrt{3}}{2} - \frac{1}{2}i \right) = 2 \left( \cos \frac{11}{6} \pi + i \sin \frac{11}{6} \pi \right)$$

$$(iii) \cos \frac{2}{3} \pi - i \sin \frac{2}{3} \pi$$

$$\cos \frac{2}{3} \pi - i \sin \frac{2}{3} \pi = -\frac{1}{2} - \frac{\sqrt{3}}{2}i = \cos \frac{4}{3} \pi + i \sin \frac{4}{3} \pi$$

$$(iv) 2 \left( \sin \frac{\pi}{3} + i \cos \frac{\pi}{3} \right)$$

$$2 \left( \sin \frac{\pi}{3} + i \cos \frac{\pi}{3} \right) = 2 \left\{ \sin \left( \frac{\pi}{2} - \frac{\pi}{6} \right) + i \cos \left( \frac{\pi}{2} - \frac{\pi}{6} \right) \right\} = 2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$