１１３．定積分①

（1）27　（2）$\dfrac{27}{2}$　（3）12　（4）$\dfrac{27}{2}$　（5）54　（6）$-\dfrac{1}{6}(\beta-\alpha)^3$

次の定積分を求めよ。

（1）$\int_0^3 (6x^2-8x+3)\,dx=\left[2x^3-4x^2+3x\right]_0^3=2\cdot 3^3-4\cdot 3^2+3\cdot 3=54-36+9=27$

（2）$\int_{-1}^2 (t^2-t+4)\,dt=\left[\dfrac{1}{3}t^3-\dfrac{1}{2}t^2+4t\right]_{-1}^2=\left(\dfrac{1}{3}\cdot 2^3-\dfrac{1}{2}\cdot 2^2+4\cdot 2\right)-\left\{\dfrac{1}{3}(-1)^3-\dfrac{1}{2}(-1)^2+4(-1)\right\}$

$=\left(\dfrac{8}{3}-2+8\right)-\left(-\dfrac{1}{3}-\dfrac{1}{2}-4\right)=\dfrac{27}{2}$

（3）$\int_{-1}^2 (-2x^2+x-3)\,dx+\int_{-1}^2 (5x^2+x+3)\,dx=\int_{-1}^2 (3x^2+2x)\,dx=\left[x^3+x^2\right]_{-1}^2=\left(2^3+2^2\right)-\left\{(-1)^3+(-1)^2\right\}$

$=8+4=12$

（4）$\int_1^3 (9x^2-2x^3)\,dx-\int_2^3 (9x^2-2x^3)\,dx=\int_1^3 (9x^2-2x^3)\,dx+\int_3^2 (9x^2-2x^3)\,dx=\int_1^2 (9x^2-2x^3)\,dx$

$=\left[3x^3-\dfrac{1}{2}x^4\right]_1^2=\left(3\cdot 2^3-\dfrac{1}{2}\cdot 2^4\right)-\left(3\cdot 1^3-\dfrac{1}{2}\cdot 1^4\right)$

$=24-8-3+\dfrac{1}{2}=\dfrac{27}{2}$

（5）$\int_{-3}^3 (4x^3+5x^2-3x-6)\,dx=2\int_0^3 (5x^2-6)\,dx=2\left[\dfrac{5}{3}x^3-6x\right]_0^3=2\left(\dfrac{5}{3}\cdot 3^3-6\cdot 3\right)=2\left(45-18\right)=54$

（6）$\int_\alpha^\beta (x-\alpha)(x-\beta)\,dx=\int_\alpha^\beta (x-\alpha)\{(x-\alpha)+(\alpha-\beta)\}\,dx=\int_\alpha^\beta \{(x-\alpha)^2-(\beta-\alpha)(x-\alpha)\}\,dx$

$=\left[\dfrac{1}{3}(x-\alpha)^3-\dfrac{\beta-\alpha}{2}(x-\alpha)\right]_\alpha^\beta=\dfrac{1}{3}(\beta-\alpha)^3-\dfrac{1}{2}(\beta-\alpha)^2$

$=-\dfrac{1}{6}(\beta-\alpha)^3$