

$$(1) 3f'(a) \quad (2) 5f'(a) \quad (3) 0 \quad (4) a^2 f'(a) - 2a f(a)$$

次の極限值を  $a$ ,  $f'(a)$  などを用いて表せ。

$$\begin{aligned} (1) \lim_{h \rightarrow 0} \frac{f(a+3h) - f(a)}{h} &= \lim_{h \rightarrow 0} \frac{f(a+3h) - f(a)}{3h} \cdot 3 \\ &= 3f'(a) \end{aligned}$$

$$\begin{aligned} (2) \lim_{h \rightarrow 0} \frac{f(a+2h) - f(a-3h)}{h} &= \lim_{h \rightarrow 0} \frac{f(a+2h) - f(a) + f(a) - f(a-3h)}{h} \\ &= \lim_{h \rightarrow 0} \left\{ \frac{f(a+2h) - f(a)}{2h} \cdot 2 + \frac{f(a) - f(a-3h)}{-3h} \cdot 3 \right\} \\ &= 2f'(a) + 3f'(a) \\ &= 5f'(a) \end{aligned}$$

$$\begin{aligned} (3) \lim_{h \rightarrow 0} \frac{f(a+h^2) - f(a)}{h} &= \lim_{h \rightarrow 0} \frac{f(a+h^2) - f(a)}{h^2} \cdot h \\ &= f'(a) \cdot 0 \\ &= 0 \end{aligned}$$

$$\begin{aligned} (4) \lim_{x \rightarrow a} \frac{a^2 f(x) - x^2 f(a)}{x-a} &= \lim_{x \rightarrow a} \frac{a^2 f(x) - a^2 f(a) + a^2 f(a) - x^2 f(a)}{x-a} \\ &= \lim_{x \rightarrow a} \frac{a^2 \{f(x) - f(a)\} - (x+a)(x-a)f(a)}{x-a} \\ &= \lim_{x \rightarrow a} \left\{ \frac{f(x) - f(a)}{x-a} a^2 - (x+a)f(a) \right\} \\ &= a^2 f'(a) - 2a f(a) \end{aligned}$$