

96. 対数の計算②

$$(1) 3 \quad (2) -5 \quad (3) 0 \quad (4) \frac{9}{2} \quad (5) \frac{2}{3} \quad (6) 2$$

次の式を計算せよ。

$$\begin{aligned} (1) \log_3 4 - \log_3 20 + 2\log_3 \sqrt{135} &= \log_3 4 - \log_3 20 + \log_3 135 \\ &= \log_3 \frac{4 \times 135}{20} \\ &= \log_3 27 \\ &= 3 \end{aligned}$$

$$\begin{aligned} (2) \log_{0.5} \frac{8}{13} - 2\log_{0.5} \frac{3}{2} + \log_{0.5} 117 &= \log_{0.5} \frac{8}{13} - \log_{0.5} \frac{9}{4} + \log_{0.5} 117 \\ &= \log_{0.5} \left(\frac{8}{13} \times \frac{4}{9} \times 117 \right) \\ &= \log_{0.5} 32 \\ &= \frac{\log_2 2^5}{\log_2 2^{-1}} \\ &= -5 \end{aligned}$$

$$\begin{aligned} (3) \log_2 12 - \log_4 24 - \frac{3}{2} \log_8 6 &= \log_2 12 - \frac{\log_2 24}{\log_2 4} - \frac{3}{2} \cdot \frac{\log_2 6}{\log_2 8} \\ &= \log_2 12 - \frac{1}{2} \log_2 24 - \frac{1}{2} \log_2 6 \\ &= \log_2 \frac{12}{\sqrt{24} \cdot \sqrt{6}} \\ &= \log_2 1 \\ &= 0 \end{aligned}$$

$$\begin{aligned} (4) (\log_2 3)(\log_7 64)(\log_{81} 343) &= \log_2 3 \cdot \frac{\log_2 64}{\log_2 7} \cdot \frac{\log_2 343}{\log_2 81} \\ &= \log_2 3 \cdot \frac{\log_2 2^6}{\log_2 7} \cdot \frac{\log_2 7^3}{\log_2 3^4} \\ &= \log_2 3 \cdot \frac{6}{\log_2 7} \cdot \frac{3 \log_2 7}{4 \log_2 3} \\ &= \frac{9}{2} \end{aligned}$$

$$\begin{aligned}
(5) \quad \left(\log_2 9 + \log_4 \frac{1}{9} \right) (\log_3 2 + \log_9 0.5) &= \left(\log_2 3^2 + \frac{\log_2 3^{-2}}{\log_2 4} \right) \left(\frac{1}{\log_2 3} + \frac{\log_2 2^{-1}}{\log_2 3^3} \right) \\
&= (2\log_2 3 - \log_2 3) \left(\frac{1}{\log_2 3} - \frac{1}{3} \cdot \frac{1}{\log_2 3^3} \right) \\
&= \log_2 3 \cdot \frac{2}{3\log_2 3} \\
&= \frac{2}{3}
\end{aligned}$$

$$\begin{aligned}
(6) \quad (\log_3 4)(\log_4 5)(\log_5 6)(\log_6 7)(\log_7 8)(\log_8 9) &= \log_3 4 \cdot \frac{\log_3 5}{\log_3 4} \cdot \frac{\log_3 6}{\log_3 5} \cdot \frac{\log_3 7}{\log_3 6} \cdot \frac{\log_3 8}{\log_3 7} \cdot \frac{\log_3 9}{\log_3 8} \\
&= \log_3 9 \\
&= 2
\end{aligned}$$