

92. 指数の拡張②

$$(1) 4 \quad (2) \frac{1}{192} \quad (3) 6 \quad (4) 8 \quad (5) 3 \quad (6) \sqrt{a} \quad (7) 1 \quad (8) \sqrt[3]{a}$$

次の式を計算せよ。

$$(1) 2^{\frac{5}{3}} \times 2^{\frac{3}{2}} \div 2^{\frac{7}{6}} = 2^{\frac{5}{3} + \frac{3}{2} - \frac{7}{6}} = 2^{\frac{10+9-7}{6}} = 2^2 = 4$$

$$(2) 4^{\frac{3}{2}} \times 27^{\frac{1}{3}} \div \sqrt{64^3} = (2^2)^{\frac{3}{2}} \times (3^3)^{\frac{1}{3}} \div \{(2^6)^3\}^{\frac{1}{2}} = 2^3 \times 3^1 \div 2^9 = \frac{1}{3 \times 2^6} = \frac{1}{192}$$

$$(3) \sqrt[4]{108} \times \sqrt[4]{12} = \sqrt[4]{108 \times 12} = \sqrt[4]{(2^2 \times 3^3) \times (2^2 \times 3)} = \sqrt[4]{2^4 \times 3^4} = 2 \times 3 = 6$$

$$(4) \left\{ \left(\frac{5}{2} \right)^{\frac{2}{3}} \right\}^{\frac{9}{2}} \div 5^{-3} = \left(\frac{5}{2} \right)^{-3} \times 5^3 = \frac{5^{-3}}{2^{-3}} \times 5^3 = 2^3 = 8$$

$$(5) (\sqrt[3]{25} + \sqrt[3]{10} + \sqrt[3]{4})(\sqrt[3]{5} - \sqrt[3]{2}) = (\sqrt[3]{5} - \sqrt[3]{2}) \left\{ (\sqrt[3]{5})^2 + \sqrt[3]{5} \cdot \sqrt[3]{2} + (\sqrt[3]{2})^2 \right\} = (\sqrt[3]{5})^3 - (\sqrt[3]{2})^3 = 5 - 2 = 3$$

$$(6) \sqrt[15]{a^7} \div \sqrt[6]{a} \times \sqrt[5]{a} = a^{\frac{7}{15}} \div a^{\frac{1}{6}} \times a^{\frac{1}{5}} = a^{\frac{7}{15} - \frac{1}{6} + \frac{1}{5}} = a^{\frac{14-5+6}{30}} = a^{\frac{1}{2}} = \sqrt{a}$$

〔別解〕

$$\sqrt[15]{a^7} \div \sqrt[6]{a} \times \sqrt[5]{a} = \sqrt[30]{a^{14}} \div \sqrt[30]{a^5} \times \sqrt[30]{a^6} = \sqrt[30]{a^{14-5+6}} = \sqrt[30]{a^{15}} = \sqrt{a}$$

$$(7) \frac{\sqrt[8]{a^3} \times \sqrt[24]{a^7}}{\sqrt[3]{a^2}} = a^{\frac{3}{8}} \times a^{\frac{7}{24}} \div a^{\frac{2}{3}} = a^{\frac{3}{8} + \frac{7}{24} - \frac{2}{3}} = a^{\frac{9+7-16}{24}} = a^0 = 1$$

$$(8) \sqrt[5]{a \times \sqrt{a \times \sqrt[3]{a}}} = \sqrt[5]{a \times \sqrt{a \times a^{\frac{1}{3}}}} = \sqrt[5]{a \times \sqrt{a^{\frac{4}{3}}}} = \sqrt[5]{a \times \left(a^{\frac{4}{3}} \right)^{\frac{1}{2}}} = \sqrt[5]{a \times a^{\frac{2}{3}}} = \sqrt[5]{a^{\frac{5}{3}}} = \left(a^{\frac{5}{3}} \right)^{\frac{1}{5}} = a^{\frac{1}{3}} = \sqrt[3]{a}$$