８３．三角関数の合成②

(1) $\sqrt{2}\cos\left(\theta-\frac{\pi}{4}\right)$	(2) $2\cos\left(\theta-\frac{5}{6}\pi\right)$	(3) $13\cos(\theta-\beta)$	(4) $\sqrt{3}\cos\left(\theta+\frac{\pi}{6}\right)$

次の式を $r\cos(\theta+\alpha)$ の形に変形せよ。

(1) $\sin\theta+\cos\theta=\sqrt{2}\left(\cos\theta\cdot\frac{1}{\sqrt{2}}+\sin\theta\cdot\frac{1}{\sqrt{2}}\right)$

$$=\sqrt{2}\left(\cos\theta\cos\frac{\pi}{4}+\sin\theta\sin\frac{\pi}{4}\right)$$

$$=\sqrt{2}\cos\left(\theta-\frac{\pi}{4}\right)$$

(2) $\sin\theta-\sqrt{3}\cos\theta=2\left\{\cos\theta\cdot\left(-\frac{\sqrt{3}}{2}\right)+\sin\theta\cdot\frac{1}{2}\right\}$

$$=2\left(\cos\theta\cos\frac{5}{6}\pi+\sin\theta\sin\frac{5}{6}\pi\right)$$

$$=2\cos\left(\theta-\frac{5}{6}\pi\right)$$

(3) $5\sin\theta+12\cos\theta=13\left(\cos\theta\cdot\frac{12}{13}+\sin\theta\cdot\frac{5}{13}\right)$

$=13\cos(\theta-\beta)$　（β は $\cos\beta=\frac{12}{13}$ かつ $\sin\beta=\frac{5}{13}$ を満たす角）

(4) $\cos\theta+\cos\left(\theta+\frac{\pi}{3}\right)=\cos\theta+\cos\theta\cos\frac{\pi}{3}-\sin\theta\sin\frac{\pi}{3}$

$$=\frac{3}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta$$

$$=\sqrt{3}\left\{\cos\theta\cdot\frac{\sqrt{3}}{2}+\sin\theta\cdot\left(-\frac{1}{2}\right)\right\}$$

$$=\sqrt{3}\left\{\cos\theta\cos\left(-\frac{\pi}{6}\right)+\sin\theta\sin\left(-\frac{\pi}{6}\right)\right\}$$

$$=\sqrt{3}\cos\left\{\theta-\left(-\frac{\pi}{6}\right)\right\}=\sqrt{3}\cos\left(\theta+\frac{\pi}{6}\right)$$