

### 83. 三角関数の合成②

$$(1) \sqrt{2} \cos\left(\theta - \frac{\pi}{4}\right) \quad (2) 2 \cos\left(\theta - \frac{5}{6}\pi\right) \quad (3) 13 \cos(\theta - \beta) \quad (4) \sqrt{3} \cos\left(\theta + \frac{\pi}{6}\right)$$

次の式を  $r \cos(\theta + \alpha)$  の形に変形せよ。

$$\begin{aligned} (1) \sin \theta + \cos \theta &= \sqrt{2} \left( \cos \theta \cdot \frac{1}{\sqrt{2}} + \sin \theta \cdot \frac{1}{\sqrt{2}} \right) \\ &= \sqrt{2} \left( \cos \theta \cos \frac{\pi}{4} + \sin \theta \sin \frac{\pi}{4} \right) \\ &= \sqrt{2} \cos\left(\theta - \frac{\pi}{4}\right) \end{aligned}$$

$$\begin{aligned} (2) \sin \theta - \sqrt{3} \cos \theta &= 2 \left\{ \cos \theta \cdot \left(-\frac{\sqrt{3}}{2}\right) + \sin \theta \cdot \frac{1}{2} \right\} \\ &= 2 \left( \cos \theta \cos \frac{5}{6}\pi + \sin \theta \sin \frac{5}{6}\pi \right) \\ &= 2 \cos\left(\theta - \frac{5}{6}\pi\right) \end{aligned}$$

$$\begin{aligned} (3) 5 \sin \theta + 12 \cos \theta &= 13 \left( \cos \theta \cdot \frac{12}{13} + \sin \theta \cdot \frac{5}{13} \right) \\ &= 13 \cos(\theta - \beta) \quad (\beta \text{ は } \cos \beta = \frac{12}{13} \text{ かつ } \sin \beta = \frac{5}{13} \text{ を満たす角}) \end{aligned}$$

$$\begin{aligned} (4) \cos \theta + \cos\left(\theta + \frac{\pi}{3}\right) &= \cos \theta + \cos \theta \cos \frac{\pi}{3} - \sin \theta \sin \frac{\pi}{3} \\ &= \frac{3}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta \\ &= \sqrt{3} \left\{ \cos \theta \cdot \frac{\sqrt{3}}{2} + \sin \theta \cdot \left(-\frac{1}{2}\right) \right\} \\ &= \sqrt{3} \left\{ \cos \theta \cos\left(-\frac{\pi}{6}\right) + \sin \theta \sin\left(-\frac{\pi}{6}\right) \right\} \\ &= \sqrt{3} \cos\left\{\theta - \left(-\frac{\pi}{6}\right)\right\} = \sqrt{3} \cos\left(\theta + \frac{\pi}{6}\right) \end{aligned}$$