７９．三角関数の加法定理①

(1)(ｉ) $\dfrac{-\sqrt{2}+\sqrt{6}}{4}$　(ⅱ) $\dfrac{\sqrt{2}+\sqrt{6}}{4}$　(ⅲ) $-2-\sqrt{3}$　(ⅳ) $\dfrac{\sqrt{2-\sqrt{2}}}{2}$　(ｖ) $\dfrac{2-\sqrt{2}}{4}$

(2)(ｉ) $\dfrac{2-2\sqrt{42}}{15}$　(ⅱ) $-\dfrac{4\sqrt{2}+\sqrt{21}}{15}$　(3) 2

次の問いに答えよ。

(1) 次の値を求めよ。

(ｉ) $\sin\dfrac{11}{12}\pi=\sin\left(\dfrac{\pi}{4}+\dfrac{2}{3}\pi\right)=\sin\dfrac{\pi}{4}\cos\dfrac{2}{3}\pi+\cos\dfrac{\pi}{4}\sin\dfrac{2}{3}\pi=\dfrac{1}{\sqrt{2}}\cdot\left(-\dfrac{1}{2}\right)+\dfrac{1}{\sqrt{2}}\cdot\dfrac{\sqrt{3}}{2}=\dfrac{-\sqrt{2}+\sqrt{6}}{4}$

(ⅱ) $\cos\dfrac{\pi}{12}=\cos\left(\dfrac{\pi}{3}-\dfrac{\pi}{4}\right)=\cos\dfrac{\pi}{3}\cos\dfrac{\pi}{4}+\sin\dfrac{\pi}{3}\sin\dfrac{\pi}{4}=\dfrac{1}{2}\cdot\dfrac{1}{\sqrt{2}}+\dfrac{\sqrt{3}}{2}\cdot\dfrac{1}{\sqrt{2}}=\dfrac{\sqrt{2}+\sqrt{6}}{4}$

(ⅲ) $\tan\dfrac{7}{12}\pi=\tan\left(\dfrac{\pi}{3}+\dfrac{\pi}{4}\right)=\dfrac{\tan\dfrac{\pi}{3}+\tan\dfrac{\pi}{4}}{1-\tan\dfrac{\pi}{3}\tan\dfrac{\pi}{4}}=\dfrac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1}=-\dfrac{\sqrt{3}+1}{\sqrt{3}-1}=-2-\sqrt{3}$

(ⅳ) $\sin\dfrac{\pi}{8}$

$\sin^2\dfrac{\pi}{8}=\dfrac{1-\cos\dfrac{\pi}{4}}{2}=\dfrac{1-\dfrac{1}{\sqrt{2}}}{2}=\dfrac{2-\sqrt{2}}{4}$

$\sin\dfrac{\pi}{8}>0$ より　$\sin\dfrac{\pi}{8}=\sqrt{\dfrac{2-\sqrt{2}}{4}}=\dfrac{\sqrt{2-\sqrt{2}}}{2}$

(ｖ) $\cos\dfrac{5}{8}\pi$

$\cos^2\dfrac{5}{8}\pi=\dfrac{1+\cos\dfrac{5}{4}\pi}{2}=\dfrac{1-\dfrac{1}{\sqrt{2}}}{2}=\dfrac{2-\sqrt{2}}{4}$

$\cos\dfrac{5}{8}\pi<0$　より　$\cos\dfrac{5}{8}\pi=-\sqrt{\dfrac{2-\sqrt{2}}{4}}=-\dfrac{\sqrt{2-\sqrt{2}}}{2}$

(2) $\dfrac{\pi}{2} < \alpha < \pi$， $0 < \beta < \dfrac{\pi}{2}$ とする。

$\sin\alpha = \dfrac{1}{3}$，$\cos\beta = \dfrac{2}{5}$ のとき，次の値を求めよ。

(ｉ) $\sin(\alpha+\beta)$

$\cos\alpha = -\dfrac{2\sqrt{2}}{3}$，$\sin\beta = \dfrac{\sqrt{21}}{5}$ であるから

$$\sin(\alpha+\beta) = \frac{1}{3}\cdot\frac{2}{5} - \frac{2\sqrt{2}}{3}\cdot\frac{\sqrt{21}}{5} = \frac{2-2\sqrt{42}}{15}$$

(ⅱ) $\cos(\alpha+\beta)$

$$= -\frac{2\sqrt{2}}{3}\cdot\frac{2}{5} - \frac{1}{3}\cdot\frac{\sqrt{21}}{5} = -\frac{4\sqrt{2}+\sqrt{21}}{15}$$

(3) $\alpha+\beta = \dfrac{\pi}{4}$ のとき，$(\tan\alpha+1)(\tan\beta+1)$ の値を求めよ。

$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ より

$\tan\dfrac{\pi}{4} = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \Leftrightarrow 1 = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \Leftrightarrow \tan\alpha\tan\beta+\tan\alpha+\tan\beta = 1$ …①

ここで，

$(\tan\alpha+1)(\tan\beta+1) = \tan\alpha\tan\beta+\tan\alpha+\tan\beta+1$ …②

であるから，①より

②$=1+1=2$