

10. 分母の有理化

$$(1) \frac{\sqrt{2}}{2} \quad (2) \sqrt{2}-1 \quad (3) 4+\sqrt{15} \quad (4) -\frac{1+3\sqrt{3}}{13} \quad (5) \frac{\sqrt{2}}{4} \quad (6) \frac{\sqrt{15}+\sqrt{6}}{3}$$

$$(1) \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

$$(2) \frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= \frac{\sqrt{2}-1}{2-1}$$

$$= \sqrt{2}-1$$

$$(3) \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{(\sqrt{5}+\sqrt{3})^2}{5-3}$$

$$= \frac{5+2\sqrt{15}+3}{2}$$

$$= \frac{8+2\sqrt{15}}{2}$$

$$= 4+\sqrt{15}$$

$$(4) \frac{\sqrt{3}-1}{2\sqrt{3}-5} = \frac{\sqrt{3}-1}{2\sqrt{3}-5} \times \frac{2\sqrt{3}+5}{2\sqrt{3}+5}$$

$$= \frac{6+5\sqrt{3}-2\sqrt{3}-5}{12-25}$$

$$= \frac{1+3\sqrt{3}}{-13}$$

$$= -\frac{1+3\sqrt{3}}{13}$$

$$\begin{aligned}
(5) \quad \frac{1}{1+\sqrt{2}-\sqrt{3}} &= \frac{1}{(1+\sqrt{2})-\sqrt{3}} \times \frac{1}{(1+\sqrt{2})+\sqrt{3}} \\
&= \frac{1}{(1+\sqrt{2})^2-3} \\
&= \frac{1}{3+2\sqrt{2}-3} \\
&= \frac{1}{2\sqrt{2}} \\
&= \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
&= \frac{\sqrt{2}}{4}
\end{aligned}$$

$$\begin{aligned}
(6) \quad \frac{\sqrt{5}+\sqrt{3}+\sqrt{2}}{\sqrt{5}+\sqrt{3}-\sqrt{2}} &= \frac{\sqrt{5}+\sqrt{3}+\sqrt{2}}{(\sqrt{5}+\sqrt{3})-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{3}+\sqrt{2}}{(\sqrt{5}+\sqrt{3})+\sqrt{2}} \\
&= \frac{(\sqrt{5}+\sqrt{3}+\sqrt{2})^2}{(\sqrt{5}+\sqrt{3})^2-2} \\
&= \frac{5+3+2+2\sqrt{15}+2\sqrt{6}+2\sqrt{10}}{8+2\sqrt{15}-2} \\
&= \frac{10+2\sqrt{15}+2\sqrt{6}+2\sqrt{10}}{6+2\sqrt{15}} \\
&= \frac{5+\sqrt{15}+\sqrt{6}+\sqrt{10}}{3+\sqrt{15}} \\
&= \frac{5+\sqrt{15}+\sqrt{6}+\sqrt{10}}{3+\sqrt{15}} \times \frac{3-\sqrt{15}}{3-\sqrt{15}} \\
&= \frac{15-5\sqrt{15}+3\sqrt{15}-15+3\sqrt{6}-3\sqrt{10}+3\sqrt{10}-5\sqrt{6}}{9-15} \\
&= \frac{-2\sqrt{15}-2\sqrt{6}}{-6} \\
&= \frac{\sqrt{15}+\sqrt{6}}{3}
\end{aligned}$$