

8. 多項式の因数分解⑤

$$(1) (a-b)(a-b-c) \quad (2) (a-b)(b-c)(c-a) \quad (3) (a+b)(b+c)(c+a)$$

$$(4) (x+y-5)(x+2y-1) \quad (5) (a+b-3)(3a-2b-2) \quad (6) (x+3y-2)(2x-y+3)$$

次の式を因数分解せよ。

$$\begin{aligned}(1) \quad a^2 + b^2 + bc - ca - 2ab &= (b-a)c + a^2 - 2ab + b^2 \\ &= -(a-b)c + (a-b)^2 \\ &= (a-b)\{-c + (a-b)\} \\ &= (a-b)(a-b-c)\end{aligned}$$

$$\begin{aligned}(2) \quad a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2) &= (-b+c)a^2 + (b^2 - c^2)a + bc^2 - b^2c \\ &= -(b-c)a^2 + (b+c)(b-c)a - bc(b-c) \\ &= (b-c)\{-a^2 + (b+c)a - bc\} \\ &= -(b-c)\{a^2 - (b+c)a + bc\} \\ &= -(b-c)(a-b)(a-c) \\ &= (a-b)(b-c)(c-a)\end{aligned}$$

$$\begin{aligned}(3) \quad (a+b+c)(ab+bc+ca) - abc &= a^2b + abc + ca^2 + ab^2 + b^2c + abc + abc + bc^2 + c^2a - abc \\ &= (b+c)a^2 + (b^2 + 2bc + c^2)a + b^2c + bc^2 \\ &= (b+c)a^2 + (b+c)^2a + bc(b+c) \\ &= (b+c)\{a^2 + (b+c)a + bc\} \\ &= (b+c)(a+b)(a+c) \\ &= (a+b)(b+c)(c+a)\end{aligned}$$

$$\begin{aligned}
 (4) \quad x^2 + 3xy + 2y^2 - 6x - 11y + 5 &= x^2 + (3y - 6)x + 2y^2 - 11y + 5 \\
 &= x^2 + (3y - 6)x + (y - 5)(2y - 1) \\
 &= x^2 + \{(y - 5) + (2y - 1)\}x + (y - 5)(2y - 1) \\
 &= (x + y - 5)(x + 2y - 1)
 \end{aligned}$$

$$\begin{array}{r}
 1_y \quad \times \quad -5 \quad \rightarrow \quad -10_y \\
 2_y \quad \times \quad -1 \quad \rightarrow \quad -1_y
 \end{array}$$

$$\begin{aligned}
 (5) \quad 3a^2 + ab - 2b^2 - 11a + 4b + 6 &= 3a^2 + (b - 11)a - 2b^2 + 4b + 6 \\
 &= 3a^2 + (b - 11)a - 2(b^2 - 2b - 3) \\
 &= 3a^2 + (b - 11)a - 2(b - 3)(b + 1) \\
 &= \{a + (b - 3)\}\{3a - 2(b + 1)\} \\
 &= (a + b - 3)(3a - 2b - 2)
 \end{aligned}$$

$$\begin{array}{r}
 1_a \quad \times \quad b - 3 \quad \rightarrow \quad (3b - 9)_a \\
 3_a \quad \times \quad -2(b + 1) \quad \rightarrow \quad (-2b - 2)_a
 \end{array}$$

$$\begin{aligned}
 (6) \quad 2x^2 + 5xy - 3y^2 - x + 11y - 6 &= 2x^2 + (5y - 1)x - 3y^2 + 11y - 6 \\
 &= 2x^2 + (5y - 1)x - (3y^2 - 11y + 6) \\
 &= 2x^2 + (5y - 1)x - (y - 3)(3y - 2) \\
 &= \{x + (3y - 2)\}\{2x - (y - 3)\} \\
 &= (x + 3y - 2)(2x - y + 3)
 \end{aligned}$$

$$\begin{array}{r}
 1_y \quad \times \quad -3 \quad \rightarrow \quad -9_y \\
 3_y \quad \times \quad -2 \quad \rightarrow \quad -2_y
 \end{array}
 \qquad
 \begin{array}{r}
 1_x \quad \times \quad 3y - 2 \quad \rightarrow \quad (6y - 4)_x \\
 2_x \quad \times \quad -(y - 3) \quad \rightarrow \quad (-y - 3)_x
 \end{array}$$